1:34
1:41
7
⇒ 25 min.

Quiz 5A, Calculus I - Calculators okay

Dr. Graham-Squire, Spring 2014

Name: Key

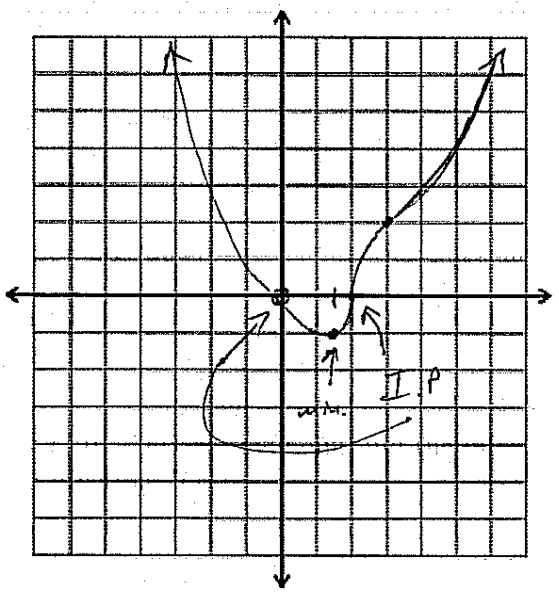
1. (5 points) You are given a function $f(x)$ such that $f'(x) = \dfrac{4x-6}{3(x-2)^{2/3}}$ and $f''(x) = \dfrac{4x-12}{9(x-2)^{5/3}}$.

   (a) Find all intervals of increase and decrease, and maximums and minimums, if there are any.

   (b) Find all intervals of concavity, and ~~any~~ inflection points, if there are any.

   (c) $f(0) = 0$. Use this point and the information from parts (a) and (b) to sketch a graph of the function $f(x)$.

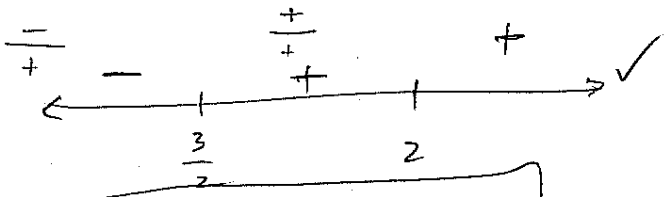
   Note: you do not need to take any derivatives for this question!



(a) $f'(x)$ dne @ $x = 2$ ✓

$f'(x) = 0$ @ $0 = 4x - 6$

$x = \frac{3}{2}$



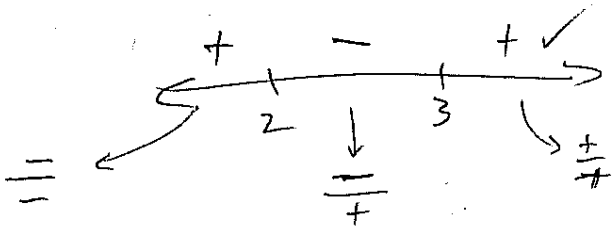
decrease: $(-\infty, 3/2)$

increase: $(\frac{3}{2}, 2), (2, \infty)$

min @ $x = 3/2$ ✓

(b) $f''(x)$ dne @ $x = 2$

$f''(x) = 0$ @ $4x - 12 = 0$ ⇒ $x = 3$



conc. up: $(-\infty, 2), (3, \infty)$

conc. down: $(2, 3)$

inf. points @ $x = 2$, $x = 3$ ✓

2. (2 points) Use calculus to evaluate the limit. If the limit does not exist, write ∞, $-\infty$, or DNE, whatever is most appropriate.

$$\lim_{x\to(-\infty)} \frac{e^{-x}}{x^2}$$

$$\lim_{x\to(-\infty)} \frac{e^{-x}}{x^2} \rightsquigarrow \frac{\infty}{\infty}$$

$$\overset{L}{=} \lim_{x\to(-\infty)} \frac{-e^{-x}}{2x} \rightsquigarrow \frac{\infty}{\infty}$$

$$\overset{L}{=} \lim_{x\to(-\infty)} \frac{e^{-x}}{2} = \frac{e^{\infty}}{2} = \boxed{\infty}$$

3. (3 points) You are given 1000 cm^2 of material to make a cylinder. Find the radius you would want in order to make a cylinder with the *largest* possible volume. The formulas for volume and surface area for a cylinder are $V = \pi r^2 h$ and $SA = 2\pi r^2 + 2\pi rh$.

$$1000 = 2\pi r^2 + 2\pi rh \implies \frac{1000 - 2\pi r^2}{2\pi r} = \frac{2\pi rh}{2\pi r}$$

$$h = \frac{1000 - 2\pi r^2}{2\pi r}$$

$$V = \pi r^2 h$$

$$V(r) = \pi r^2 \left(\frac{1000 - 2\pi r^2}{2\pi r}\right)$$

$$V(r) = \frac{1000}{2} r - \frac{2\pi r^3}{2}$$

$$V(r) = 500r - \pi r^3$$

$$V'(r) = 500 - 3\pi r^2$$

$$0 = 500 - 3\pi r^2$$

$$3\pi r^2 = 500$$

$$\sqrt{r^2} = \sqrt{\frac{500}{3\pi}}$$

$$r = 7.28$$

Round to nearest 0.01